

MATH 42-NUMBER THEORY
EXTRA CREDIT ASSIGNMENT
DUE MAY 10, 2011

This problem set is extra credit, due on the last day of class. You may turn it in earlier if you wish. Because these problems are somewhat open-ended, you are encouraged to work in groups. However, if you work in a group, you must write up solutions individually and say who you worked with.

Thank you to Patrick Cox for coming up with the conjecture that led to this problem set!

Consider U_p where p is prime. We know that U_p has $\varphi(p-1)$ generators, but what is the sum of these generators? For example, in U_7 , the generators are 3 and 5, and $3+5 \equiv 1 \pmod{7}$. Patrick's conjecture is that the sum of the generators is always either -1 , 0 or $1 \pmod{p}$.

1. If g is a generator mod p , is $-g$ necessarily a generator? Give a general statement about when $-g$ is a generator (i.e. for which primes p), and prove it.
2. Give a general statement about when the sum of the generators mod p is 0 , -1 , or 1 . (Hint: Look at the factorization of $p-1$.)
3. Prove your statement from problem 2. Partial credit will be awarded for proofs in special cases.